Chapter Three Semantic Problems A1:
WebCT Discussion

Assignment: **translate** the following English argument into formal language; then build a **truth table** to decide if the argument is **valid**.

Translation Table:

> **P**: We will have a tax cut.
> **Q**: We will have increased spending on Logic research.

Argument:

> 1. We won't have *both* a tax cut *and* increased spending on Logic research.
> 2. We won't have increased spending on logic research unless we have a tax cut.
> (so,) We won't have increased spending on Logic research, but we will have a tax cut.

Discussion: We begin translating the first premise by putting in sentence letters for each of its subject matter sentences.

> **P**: <u>We will have</u> a tax cut.
> **Q**: <u>We will have</u> increased spending on Logic research.

1. <u>We wo</u>n't <u>have</u> *both* a tax cut *and* increased spending on Logic research.
   n't *both* P *and* Q

(Note that "<u>We will have</u>," which shows up in both subject matter sentences, is pulled to the front in Sentence (1), and just stated once.)

"Both… and" is translated by the wedge (with its matching parentheses).

1. We won't have *both* a tax cut *and* increased spending on Logic research.
   n't $(P \wedge Q)$

The sentence as a whole is the denial of this "*both… and*" claim – hence a negation. "N't" is translated by the tilde.

P: We will have a tax cut.
Q: We will have increased spending on Logic research.

1. We won't have *both* a tax cut *and* increased spending on Logic research.
   $\sim (P \wedge Q)$

We likewise put in sentence letters for the second premise.

P: We will have a tax cut.
Q: We will have increased spending on Logic research.

2. We won't have increased spending on logic research unless we have a tax cut.
   n't Q unless P

This is an "unless" sentence, with "n't Q" as its left part, and "P" as its right part. "*Unless*" is translated by the vel (with its matching parentheses).

2. We won't have increased spending on logic research unless we have a tax cut.
   (n't Q $\vee$ P)

"N't" is translated by the tilde.

2. We won't have increased spending on logic research unless we have a tax cut.
   $(\sim Q \vee P)$

Finally, we replace subject matter sentences with sentence letters in the conclusion.

P: We will have a tax cut.
Q: We will have increased spending on Logic research.

∴ We won't have increased spending on Logic research, but we will have a tax cut.
n't Q, but P

The comma break points to "*but*" as the main connective of this English sentence. "*But*" is translated by the wedge (with its matching parentheses).

∴ We won't have increased spending on Logic research, but we will have a tax cut.
(n't Q $\wedge$ P)

And "n't" is translated by the tilde.

∴ We won't have increased spending on Logic research, but we will have a tax cut.
($\sim$Q $\wedge$ P)

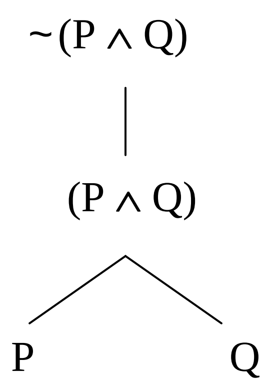
The logical form of the whole argument is thus as follows.

1. $\sim(P \wedge Q)$
2. $(\sim Q \vee P)$

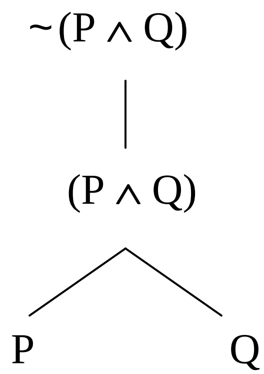
∴ $(\sim Q \wedge P)$

Next we build a truth table for this formal argument, to test it for validity. We build a truth table for each premise, and for the conclusion.

If you are unsure how to build a truth table for a sentence, its construction tree is a reliable guide. The construction tree for the first premise is as follows.



The truth table for this first premise will feature just these four sentences, in this order (starting from the bottom, with the sentence letters).



P	Q	(P ∧ Q)	~(P ∧ Q)

For two sentence letters, we need four valuations. We build valuations for the sentence letters “P” and “Q” in the usual way.

P	Q	(P ∧ Q)	~(P ∧ Q)
1	1		
1	0		
0	1		
0	0		

The sentence "(P ∧ Q)" is a conjunction, and so follows the semantic rule for conjunctions.

Conjunction Rule:

●	▲	(● ∧ ▲)
1	1	1
1	0	0
0	1	0
0	0	0

This conjunction is true when both its parts are true – Valuation 1. The conjunction is false in the other valuations.

P	Q	(P ∧ Q)	~(P ∧ Q)
1	1	**1**	
1	0	**0**	
0	1	**0**	
0	0	**0**	

Finally, "~(P ∧ Q)" is the negation of "(P ∧ Q)," and so (like all negations) follows the Negation Rule of our semantics.

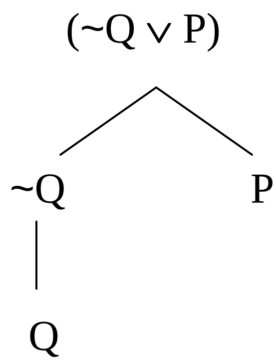
Negation Rule:

●	~●
1	0
0	1

When "(P ∧ Q)" is true (Valuation 1), "~(P ∧ Q)" is false. When "(P ∧ Q)" is false (Valuations 2, 3, and 4), "~(P ∧ Q)" is true.

P	Q	(P ∧ Q)	~(P ∧ Q)
1	1	1	**0**
1	0	0	**1**
0	1	0	**1**
0	0	0	**1**

As a construction tree shows, the truth table for the *second premise* requires the following parts.



Two of these sentences – “Q” and “P” –already appear in our truth table.

P	**Q**	(P ∧ Q)	~(P ∧ Q)
1	1	1	0
1	0	0	1
0	1	0	1
0	0	0	1

So we don’t need to build them again. We just need to add the two remaining sentences, “~Q” and “(~Q ∨ P)”.

P	Q	(P ∧ Q)	~(P ∧ Q)	~Q	(~Q ∨ P)
1	1	1	0		
1	0	0	1		
0	1	0	1		
0	0	0	1		

"~Q" is the negation of "Q," and so follows the Negation Rule: when "Q" is true (Valuations 1 and 3), "~Q" is false. And when "Q" is false (Valuations 2 and 4), "~Q" is true.

Negation Rule:

●	~●
1	0
0	1

P	Q	(P ∧ Q)	~(P ∧ Q)	~Q	(~Q ∨ P)
1	1	1	0	0	
1	0	0	1	1	
0	1	0	1	0	
0	0	0	1	1	

"(~Q ∨ P)" is a disjunction, with "~Q" as its left part and "P" s its right part. It follows the Disjunction Rule.

Disjunction Rule:

●	▲	(● ∨ ▲)
1	1	1
1	0	1
0	1	1
0	0	0

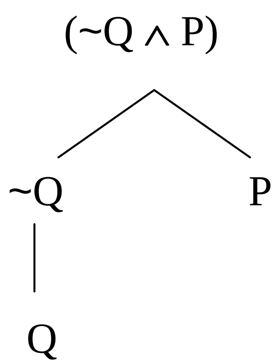
A disjunction is only false when both its parts are false. The only Valuation where both parts – "~Q" and "P" – are false, is Valuation 3. So the sentence is false in Valuation 3.

P	Q	(P ∧ Q)	~(P ∧ Q)	~Q	(~Q ∨ P)
1	1	1	0	0	
1	0	0	1	1	
0	1	0	1	**0**	**0**
0	0	0	1	1	

It’s true in the other three valuations.

P	Q	(P ∧ Q)	~(P ∧ Q)	~Q	(~Q ∨ P)
1	1	1	0	0	**1**
1	0	0	1	1	**1**
0	1	0	1	0	0
0	0	0	1	1	**1**

Finally, we build a truth table for the conclusion. A construction tree shows that the truth table calls for these parts.



Note that here the first three parts of the tree – “Q,” “P.” and “~Q” – are already contained in our truth table as it stands.

P	Q	(P ∧ Q)	~(P ∧ Q)	~Q	(~Q ∨ P)
1	1	1	0	0	1
1	0	0	1	1	1
0	1	0	1	0	0
0	0	0	1	1	1

So we only need to add the final part of the tree – the sentence “(~Q ∧ P)” itself – to complete our truth table for this argument.

P	Q	(P ∧ Q)	~(P ∧ Q)	~Q	(~Q ∨ P)	**(~Q ∧ P)**
1	1	1	0	0	1	
1	0	0	1	1	1	
0	1	0	1	0	0	
0	0	0	1	1	1	

"(~Q ∧ P)" is a conjunction, and so follows the Conjunction Rule.

Conjunction Rule:

●	▲	(● ∧ ▲)
1	1	1
1	0	0
0	1	0
0	0	0

"(~Q ∧ P)" is only true when both its parts are true. The only valuation here where both "~Q" and "P" are true, is Valuation 2.

P	Q	(P ∧ Q)	~(P ∧ Q)	~Q	(~Q ∨ P)	(~Q ∧ P)
1	1	1	0	0	1	
1	0	0	1	**1**	1	**1**
0	1	0	1	0	0	
0	0	0	1	1	1	

In the other valuations, "(~Q ∧ P)" is false.

P	Q	(P ∧ Q)	~(P ∧ Q)	~Q	(~Q ∨ P)	(~Q ∧ P)
1	1	1	0	0	1	**0**
1	0	0	1	1	1	1
0	1	0	1	0	0	**0**
0	0	0	1	1	1	**0**

Our final task is to decide, on the basis of this truth table, whether the argument is valid or invalid.

To decide this, we first pick out **the valuations** where **all the premises** are **true**. Here there are two valuations that make both Premise (1) and Premise (2) true: Valuations 2 and 4.

			(1)		(2)	$\therefore$
P	Q	$(P \wedge Q)$	$\sim(P \wedge Q)$	$\sim Q$	$(\sim Q \vee P)$	$(\sim Q \wedge P)$
1	1	1	0	0	1	0
1	0	0	**1**	1	**1**	1
0	1	0	1	0	0	0
0	0	0	**1**	1	**1**	0

Next we check whether the conclusion is true in each such valuation. We see that the conclusion is true in Valuation 2.

			(1)		(2)	$\therefore$
P	Q	$(P \wedge Q)$	$\sim(P \wedge Q)$	$\sim Q$	$(\sim Q \vee P)$	$(\sim Q \wedge P)$
1	1	1	0	0	1	0
1	0	0	**1**	1	**1**	**1**
0	1	0	1	0	0	0
0	0	0	1	1	1	0

But don't celebrate yet: to be valid, the conclusion must be true in every possible situation where the premises are (all) true. Alas, the conclusion is *not* true in Valuation 4.

			(1)		(2)	$\therefore$
P	Q	$(P \wedge Q)$	$\sim(P \wedge Q)$	$\sim Q$	$(\sim Q \vee P)$	$(\sim Q \wedge P)$
1	1	1	0	0	1	0
1	0	0	1	1	1	1
0	1	0	1	0	0	0
0	0	0	**1**	1	**1**	**0**

What we find in Valuation 4 is a possible situation (valuation) where all the premises of the argument are true, but the conclusion is still false. Valuation 4 is thus a **validity counterexample** for this argument.

				(1)		(2)	∴
	P	Q	$(P \wedge Q)$	$\sim(P \wedge Q)$	$\sim Q$	$(\sim Q \vee P)$	$(\sim Q \wedge P)$
	1	1	1	0	0	1	0
	1	0	0	1	1	1	1
	0	1	0	1	0	0	0
⇨	0	0	0	**1**	1	**1**	**0**

Since our search through all the different possibilities (valuations) turned up a validity counterexample, we know that this argument is **invalid**. (The conclusion does *not* follow from the premises.)

1. We won't have *both* a tax cut *and* increased spending on Logic research.
2. We won't have increased spending on logic research unless we have a tax cut.

∴ We won't have increased spending on Logic research, but we will have a tax cut.

1. $\sim(P \wedge Q)$
2. $(\sim Q \vee P)$

∴ $(\sim Q \wedge P)$

Verdict: **Invalid** argument

Moral of the story: maybe you couldn't just look at that English argument and decide whether or not the conclusion follows from the premises – maybe the English argument was too complicated to juggle in your imagination. No matter: the formal test of validity gives us a verdict, no matter how complicated the argument is.